A PROPOSED APPROACH TO CHANGE OF SUPPORT CORRECTION FOR MULTIPLE INDICATOR KRIGING, BASED ON *P*-FIELD SIMULATION

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Abstract

While there have been significant improvements in estimation techniques, ways of addressing the "difficult problem" of support effect have languished. For geostatisticians working in mineral resource estimation this issue is critical. There is now widespread industry acceptance that mining optimisation studies are best built on resource models that predict the tonnes and grade that will be identified during mining. Ore loss and dilution vary with the scale of mining, notionally represented by the size of the selective mining unit (SMU). Recoverable resource models, i.e. those incorporating change of support correction, are based on implicit assumptions. The various approaches used in disjunctive kriging (DK), multiple indicator kriging (MIK), probability kriging (PK) and multigaussian approach (MG) are discussed.

A proposed alternative to the adoption of parametric change of support correction factors is presented where the local correction factor is deduced empirically. The approach is based on characterisation of the short-scale variability within the domain of interest. The simulation concept is borrowed from MG but with the conditioning moments replaced by the MIK cdf. The distributions of the expected SMU sized values (grades) are thus derived empirically. This proposed approach, referred to here as p-field correction for multiple indicator kriging (pMIK), is expected to allow the local impact of selective mining or bulk mining to be characterised.

Key Words: *geostatistics, non-linear estimation, multiple indicator kriging, change of support.*

Introduction

Point support and block support

There have been significant improvements in estimation techniques with the introduction first of linear methods (e.g. simple kriging or ordinary kriging) and subsequently non-linear methods (e.g. indicator kriging). These methods have proved to be significant advances over the classical methods of weighted averaging to interpolate values at some locations between known data. From the outset, the primary advantages of kriging were recognised as being that it is an unbiased estimator and that it minimises the error of estimation. This is achieved by combining information about the directional correlation of the data with the configuration of the data around the unknown location. The variogram is a popular tool for quantifying this spatial relationship. Using a kriging plan, the appropriate weights are allocated to the surrounding data to enable the unknown grade to be interpolated.

The work originally done by D. G. Krige in South Africa (Krige, 1951, 1962) was primarily concerned with selective mining of gold to a cut-off grade and changes to the distribution of values as their support (size) increased from samples to ore blocks. The impact of correlation and regression on the grade-tonnage curve was recognised by Krige and has become embodied in expressions referred to as Krige's relationship. This may be summarised as "the total variance is equal to the sum of the within block variance and the between block variance" (David, 1977, p. 98). This relationship provides a theoretical basis for the application of a change of support correction, so that estimation based on a point support (e.g. 1 m composited drill sample assays) can be used to represent information on a larger support (e.g. ore blocks).

Resource estimation

One of the major difficulties for classical methods of resource estimation is this issue of support. Polygonal estimates can be biased if the drill hole samples (the point support data) are each allocated a specific volume of influence. No matter how this volume is assigned, the assumption that the point estimates correctly represent grades for such volumes is demonstrably and intuitively incorrect. The variance of sample grades and volume (block) grades would be the same, contrary to Krige's relationship. The differences in grade expected for a series of large blocks (e.g. stoping blocks or mining benches) will be much less than that seen in the drill samples. Similarly the maximum grade of large blocks will be much less than the maximum grade of the samples.

At the resource estimation stage the sampling data set is relatively sparse compared to that finally available from grade control. Krige noticed that estimates made using such sparse data were conditionally biased due to the smoothing effect of estimation. Nevertheless the objective in resource estimation is to predict what will happen during mining. We are unable to accurately define the positions of the smaller ore parcels that will be defined by grade control. Frequently we only have the sparse data. Let us assume that the grade control data will have the same distribution characteristics as the resource sampling data, but is dispersed (or "lost") within the deposit. Models can then be constructed to indicate the probability of values above a threshold that will be identified (or "found") assuming a nominal selective mining unit (SMU) size.

Thus a two step process is required: estimation of the local distribution and then correction of that distribution to represent the distribution of SMU blocks. Non-linear geostatistics addresses estimation of the point support local distributions. There are various strategies that must then be used to adjust the point support distribution to that expected for blocks of a nominated volume. These strategies are collectively referred to here as change of support corrections (Isaaks and Srivastava, 1989, pp. 458-488). Such corrections, e.g. adjusting one distribution to another while keeping the means the same, involve making assumptions which are examined further in this paper.

Recoverable resource models

There has been a significant rise in the sophistication of resource estimates being carried out on orebodies, with the successful application of kriging to many deposits previously believed intractable. Recent published examples include the use of multiple indicator kriging for gold deposits (Gaze, *et al*, 1997), iron ore deposits (Collings, *et al*, 1997), and nickel (Lipton *et al*, 1998). Based on the successful performance of probabilistic resource models at a number of operations, the mining industry is gaining confidence in such computer intensive methods.

By comparison with the sophistication of kriging techniques being applied, ways of addressing the support effect have languished.

The mining resolution is notionally represented by the size of the selective mining unit (SMU). This may be regarded as the average minimum dimensions for a discrete ore block. In many cases ore blocks exceed these dimensions. Where this happens, assumptions that depend on the SMU block size have less impact. As the mining block size increases, the mining scale changes from selective mining to bulk mining. The low costs provided by economies of scale must be balanced against the lower mining head grade as dilution increases. Mines that have difficulty with dilution are frequently found to have chosen too small an SMU size in building the resource model. In striving to demonstrate a high head grade, such models overestimate the ability of the proposed mining fleet to mine selectively and achieve tonnage targets.

The precision and resolution of the grade control sampling (the information effect) may be regarded as part of the mining selectivity process for the subsequent discussion.

Current practices

Change of support correction

Linear estimation (e.g. polygonal, inverse distance, and kriging) methods produce a single outcome for each location. The estimation error depends on data density. Estimation of average grades for block sizes approximating the nominal SMU size would require a comparable sampling data density (resolution). Such information is usually only available at the grade control stage. Although we are unable to accurately define the positions of smaller units, it is possible to forecast on a probabilistic basis, over an area suited to the exploration data density, the proportion of SMU size blocks that will be selected above a nominated cut-off grade. Non-linear estimation methods were designed to address this support issue.

A brief survey follows of how the change of support is currently addressed in recoverable resource estimation. Emphasis here is on declaring the implicit assumptions rather than on a mathematical exposition of the various methods, which are provided in the references. The approaches used in disjunctive kriging, multiple indicator kriging, probability kriging and the multigaussian approach are discussed. More detailed comparative reviews of these methods are provided by Marechal (1984); Journel (1985); and Knudsen and Baafi (1987). This discussion provides the context for a proposed new approach to change of support correction.

Disjunctive kriging (DK)

The DK method (Matheron, 1976) was developed to predict the recovery of material at different cut-off grades from the distribution of sample values. It does this by modelling the density function of the variable of interest (e.g. grade) from all the available data. The sample data set is transformed to the standard normal distribution using a limited expansion of Hermite polynomials (see Journel and Huijbregts, 1978, p.573-580). The solution to these equations is derived using numerical integration and the model is checked against the original data set for goodness of fit. Following the normalisation, structural analysis (variography) is performed and the process of disjunctive kriging is carried out.

Using DK there are a number of approaches available for change of support correction. For example in the discrete Gaussian model, the process of back transformation (anamorphosis) may be accompanied by a change of support coefficient r. This is determined by assuming that the means are the same, the variance of the points is derived from the variogram and the points are randomly distributed inside the blocks (Rivoirard, 1994, p.81).

The covariances of the points (x–x) and the blocks (v–v) are related by:

$$\rho_{x_i x_j} = r^2 \rho_{v_i v_j}$$

The mathematics of DK is complex and the numerical integration process may suffer from unsuccessful convergence specifically related to the definition of the extreme tails of the experimental distribution. Rivoirard (1994, p.90) describes it as a neat and yet mathematically consistent model, applicable under various distribution assumptions to local estimation and simulation. Deutsch and Journel (1992, p83) note that the DK method is a slight generalisation of the bivariate Gaussian model and "due to the problem of which transfer function to retain … one may be better off using the most congenial and parsimonious of all RF models, the multivariate Gaussian model".

Multiple indicator kriging (MIK)

The application of an indicator function as a threshold value (e.g. a cut-off grade) to the sample data set enables the data values to be transformed to a set of indicators, so that the data set is now a set of 0 and 1 values (e.g. the indicator 1 is assigned if the sample data value is equal to or above the cut-off grade, otherwise the indicator 0 is assigned). This is a non-linear transformation in that linear estimates (whether arithmetic averaging or kriging) carried out on the transformed values do not retain the original sample data relationships.

Kriging the indicators for a given location produces a result between 0 and 1 which cannot be transformed back to a sample grade. Rather this kriged indicator value must be regarded as a pseudo-probability (*p*-value) of exceeding the threshold (cut-off grade) at that location. Multiple indicator kriging (MIK) involves the determination of indicators at a number of thresholds so that the cdf model of *p*-values is available for each location. To define the distribution of blocks of a nominal SMU size, the variance of the cdf models must be reduced. This post-processing of the MIK inventory is frequently done using either the affine correction or the indirect lognormal correction.
The affine correction (Isaacs and Srivastava, 1989, p.471) is the more simple approach. The sample distribution is symmetrically squeezed about the mean value to reduce the variance. This reduces the high grades but increases the low grades to preserve the shape of the distribution. For gold deposits with strong positive skew, or iron deposits with a negative skew and a maximum grade (e.g. 69.4 % Fe for hematite), this may not be appropriate. Rossi and Parker (1993) note additional difficulties caused by a high nugget to sill ratio for variograms.

The indirect lognormal correction (Isaacs and Srivastava, 1989, p.472) assumes for transformation that the point distributions and the inferred block distributions are both lognormal. The values are transformed to reduce the variance and then they are rescaled to correct the mean. The extent of the rescaling required depends on how well the assumption of lognormality holds for the point distributions. This results in asymmetric squeezing of the distribution about the mean, reducing the skewness of the distribution.
The post-processing of the MIK point support model is carried out to define a block support model that predicts the mining resolution expected using a proposed SMU size. After obtaining the SMU distribution, application of a cut-off grade allows the probability of exceeding that cut-off to be predicted. The model can be interrogated to define the probability (or its inverse, the risk) of achieving a tonnage and grade. Probabilities of exceeding thresholds may be contoured to demonstrate the spatial relationships in the resource model, and the grade can be reported above nominated cut-off grades. Journel (1985, p.565) commends this method for its simplicity and robustness, noting that it is distribution free.

Some significant points to be aware of are:

- Reporting the tonnes and grades from such models requires a clear understanding of the *p*-values and associated grade estimates.
- The resulting model is assumed to define the recoverable resource by including planned dilution, i.e. the impact of the effective SMU size. This relates to the grade control data support (both precision and resolution) and the mining practices. There is no theoretical or parametric demonstration that the change of support correction has correctly modelled these effects. This can be demonstrated using conditional simulation and is strongly recommended.

Probability kriging (PK)

Probability or proportion kriging (PK) is an enhancement of indicator kriging (IK) described by Sullivan (1984). The IK method uses only the indicator data whereas PK uses the grade information in addition to the indicator values through a co-kriging process. Since the grade values and the indicator values are of different orders of magnitude, use of both in the same system may give numerical problems. Consequently the data values are transformed to a uniform distribution, sorting them by increasing grade and assigning them their cumulative frequency (i.e. a rank order transform). The cdf is then estimated by solving a co-kriging system involving both the indicators and the rank order transformed data. The PK approach requires calculation of the indicator variograms, the rank order transformed (grade) variograms and the cross variograms between these two variables.

Journel (1985) notes: “The PK approach retains the essential characteristics of simplicity and robustness of the IK approach.” However, despite the additional complexity and information retained, probability kriging does not alter the requirement for a change of support correction. Again the affine correction or the indirect lognormal correction may be applied to the point support distributions.

The multigaussian approach (MG)

Unlike PK and IK, which are nearly free of major probabilistic interpretations or hypotheses, the multigaussian approach (MG) of Verly (1984) is highly dependent on the probabilistic interpretation of the grade values and distribution hypotheses.

The sample values are transformed to a standard normal distribution. Variograms are produced and kriging is carried out followed by a Monte Carlo simulation to solve the integration required for the change of support. An underlying assumption is that the normal score transforms are multivariate normally distributed, i.e. that all the inter-relationships between the variable of interest, at all thresholds, are normally distributed. As with lognormal kriging (a special case of MG where the lognormal transform is used instead of the Gaussian transform) local errors in the estimation of the variance of the cdf may be enhanced. This can have the unfortunate consequence of creating ore in regions of consistent low grade where the data set is sparse.

Journel (1985) notes: “The MG approach stands out by its extreme rigour and consistency. A single, albeit strong, hypothesis is made in the beginning from which all subsequent results are deduced without any further hypothesis or approximation. Also, the MG approach is presently the only approach available that can handle the difficult problem of support effect with strict consistency. This does not necessarily mean that it would provide the most accurate solution.”

Alternative methods

An implicit assumption in the approaches outlined previously for change of support correction is that the blocks are selected as ore or waste independently. This ignores the spatial relationship inherent in mining data. In addition each of the methods makes specific assumptions regarding distributions and their relationships through transforms. Change of support correction for the non-linear approaches discussed (DK, MIK, PK, and MG) all suffer from a subjective assumption of the appropriateness of a parametric distribution model. An alternative to the adoption of parametric change of support correction factors is required.

It can be safely assumed that ore grades average arithmetically, the mean grade of points and blocks does not change, and that the variance of block grades reduces; however these assumptions are not enough (Rossi and Parker, 1993). A number of pragmatic solutions were presented by Rossi, Parker and Roditis (1993) including using conditional simulation to provide a training model to define the change of support correction, modelling of grade control sampling, modifying the kriging method using grade thresholds and/or adjusting the kriging plan to produce block estimates with the theoretically appropriate dispersion variance. These methods are all made possible or enhanced by the availability of some short spaced data for the orebody.

Using conditional simulation to provide a training model to deduce the change of support correction factor has the advantage that the result is deduced empirically rather than parametrically. However the application of a local training set to the global resource model raises questions of representativity and stationarity. Conditional simulation approaches attempt to determine the local SMU cdf empirically. Given good quality data (always the ultimate limitation) the empirical method frequently provides more realistic solutions than parametric modelling to geological and mining problems.

There is now an expectation amongst geostatisticians that comprehensive conditional simulation models can be produced for recoverable resource estimation. Unfortunately, with many methods of simulation the computation time increases exponentially as the volume to be simulated increases. Add to this the massive data storage requirements for a large number of realisations (or the definition of many ccdf models) and other more efficient solutions still look attractive.

The proposed approach

By using the Gaussian kriged values as conditional moments for a simulation process, the MG model suggests a different approach to Journel's "difficult problem" of support correction. However the MG approach is parametric with a strong multi-normal hypothesis.

The proposed approach is to borrow the simulation concept from MG but with the conditioning moments replaced by the MIK cdf. This has been developed and trialled following successful experience with probability field simulation (***p*-field simulation**) models to define variability (Khosrowshahi and Shaw, 1997). The *p*-field approach (Srivastava, 1992) creates non-conditional simulated realisations which reflect the deposit variability, and are then conditioned to the local cdf estimated by MIK.

This *p*-field correction for multiple indicator kriging (*p*MIK) allows the impact of selective mining and bulk mining to be assessed for the deposit of interest. The method has been found to be sensitive to the number of simulation realisations and the resolution of discretisation used for the simulation model. However the speed of the *p*-field algorithm, and the requirement to only retain the local ccdf models for the nominated SMU size, means that a large number of realisations are manageable.

The use of local conditioning overcomes another implicit assumption in all previous change of support corrections. The problem of "adaptive geometry" (Rivoirard, 1994; Rossi, Parker and Roditis, 1993) occurs where the nominal SMU size bears no relationship to the real spatial continuity of mined ore blocks. Many ore zones of contiguous mineralisation are much bigger than the nominal SMU and the assumption that SMU sized blocks are all independent is just not true in practice. The *p*MIK approach compensates for this by producing contiguous SMU blocks with similar values where there is good continuity. In areas where the MIK model defines a cdf with high variance, the effect of local dilution incurred for a nominal SMU size is significantly increased.

It must be remembered that there may be causes for not achieving the predicted recoverable grade that are not addressed by this proposed method. Examples that lead to additional imprecision during mining include:

- Imprecision or bias of the grade control assay results at the cut-off grade,
- Vertical heave and lateral movements due to blasting,
- Insufficient survey control of sample locations relative to dig lines, and
- Inappropriate application of visual control for selection of ore.

The impact of grade control procedures on dilution, unseen ore loss and mining recovery has been examined in other forums (e.g. Shaw and Khosrowshahi, 1997).
Despite the many difficulties in predicting the recoverable resource estimate, significant improvements can be achieved by addressing issues one at a time. The first step is to use good estimation methods that include an appropriate change of support correction based on local data.

The proposed *p*MIK method is expected to provide an effective way to achieve empirical correction and to be an improvement on parametric based methods. This approach is currently still under development.

References

Collings, P. S., Khosrowshahi, S. and Ness, P. K., 1997. Geological modelling and geostatistical resource estimation of the Hope North Deposit. *Ironmaking Resources and Reserves Estimation*, Perth, Australasian Institute of Mining and Metallurgy, pp.105 - 116

David, M., 1977. *Geostatistical ore reserve estimation*. Pub. Elsevier, Amsterdam, 364pp.

Deutsch, C. V. and Journel, A. G., 1992. *GSLIB geostatistical software library and user's guide*. Oxford University Press, Inc, New York, 340 pp.

Gaze, R. L., Khosrowshahi, S., Gibbs, D., and Grove, A., 1997. Geological and geostatistical resource estimation of the Cleo Deposit, Sunrise Dam – a balanced approach. *Gold & Nickel Ore Reserve Estimation Practice*, Towards 2000

Australasian Institute of Mining and Metallurgy Mineral Resources And Ore Reserves Seminars, Australasian Institute of Mining and Metallurgy, Kalgoorlie Branch, WA, 21 October, pp121-139.

Isaacs, E. H. and Srivastava, R. M., 1989. *Applied geostatistics*, Oxford University Press, 561 pp.

Journel, A. G., and Huijbregts, C. J., 1978. *Mining geostatistics*, Academic Press, London, 600pp.

Journel, A.G., 1985. Recoverable reserves estimation - the geostatistical approach. *Mining Engineering*, pp.563-568, June.

Khosrowshahi, S. and Shaw, W. J., 1997. Conditional simulation for resource estimation and grade control – principles and practice. *Proceedings of the World Gold 97 Conference, Singapore*, Australasian Institute of Mining and Metallurgy, pp.275-282.

Knudsen, H. P. and Baafi, E.Y., 1987. Indicator kriging and other new geostatistical tools. *Proceedings of the Pacific Rim Congress*, Australasian Institute of Mining and Metallurgy, pp. 859-864.

Krige, D. G., 1951. A statistical approach to some basic mine valuation problems on the Witwatersrand. *Journal of Chemical, Metallurgical and Mining Engineering Society of South Africa*, December, pp.119-139 (and following discussion during 1952).

Krige, D. G., 1962. Effective pay limits for selective mining. *Journal of the South African Institute of Mining and Metallurgy*, pp.345-363, January.

Lipton, I. T., Gaze, R. L., Horton, J. A. and Khosrowshahi, S., 1998. Practical application of multiple indicator kriging and conditional simulation to recoverable resource estimation for the Halley's lateritic nickel deposit. *Beyond Ordinary Kriging: Non-Linear Geostatistical Methods In Practice, Perth,* Geostatistical Association of Australasia, (this volume).

Marechal, A., 1984. Recovery estimation: a review of models and methods. *Verly, G. (Ed) Proceedings of 2nd Geostatistical Congress, California*, NATO ASI series, Reidel, Dordrecht (Holland), pp.385-420.

Matheron, G., 1976. A simple substitute for conditional expectation: disjunctive kriging. *Proceedings of the 1st Geostatistics Congress, Rome*, NATO ASI series, Reidel, Dordrecht (Holland), pp.221-236.

Rivoirard, J., 1994. *Introduction to disjunctive kriging and non-linear geostatistics*, Clarendon Press, Oxford, 180pp.

Rossi, M.E. and Parker, H. M., 1993. Estimating recoverable reserves: is it hopeless? *Dimitrakopoulos, R., (Ed.), Geostatistics For The Next Century*, Kluwer Academic, Boston, pp.259-276.

Rossi, M.E., Parker, H. M., and Roditis, Y. S., 1993. Evaluation of existing geostatistical models and new approaches in estimating recoverable reserves. *Proceedings of the XXIV APCOM, Montreal, Canada*.

Shaw, W. J. and Khosrowshahi, S., 1997. Grade control sampling and ore blocking: optimisation based on conditional simulation. *Third International Mining Geology Conference Proceedings, Launceston Tasmania*, Australasian Institute of Mining and Metallurgy and Australian Institute of Geoscientists, pp.131-134.

Srivastava, R. M., 1992. Reservoir characterisation with probability field simulation. *Society of Petroleum Engineers Annual Conference*, Washington, D.C., pp. 927-938.

Sullivan, J., 1984. Conditional recovery estimation through probability kriging. Theory and practice. *Verly, G. (Ed) Proceedings of 2nd Geostatistical Congress, California*, NATO ASI series, Reidel, Dordrecht (Holland), pp.365-384.

Verly, G., 1985. The block distribution given a point multivariate normal distribution. *Verly, G. (Ed) Proceedings of 2nd Geostatistical Congress, California*, NATO ASI series, Reidel, Dordrecht (Holland), pp.495-515.

Verly, G. and Sullivan, J., 1985. Multigaussian and probability krigings - application to the Jerritt Canyon deposit. *Mining Engineering*, pp.568-574, June.