BEYOND ORDINARY KRIGING — AN OVERVIEW OF NON-LINEAR ESTIMATION

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Abstract

Many geostatistical variables have sample distributions that are highly positively skewed. Because of this, significant deskewing of the histogram and reduction of variance occurs when going from sample to block support, where blocks are of larger volume than samples. When making estimates in both mining and non-mining applications we often wish to map the spatial distribution on the basis of block support rather than sample support. The SMU or selective mining unit in mining geostatistics refers to the minimum support upon which decisions (traditionally: ore/waste allocation decisions) can be made. The SMU is usually significantly smaller than the sampling grid dimensions, in particular at exploration/feasibility stages. Linear estimation of such small blocks (for example by inverse distance weighting – IDW – or ordinary Kriging – OK) results in very high estimation variances, i.e. the small block linear estimates have very low precision. A potentially serious consequence of the "small block linear estimation" approach is that the grade-tonnage curves are distorted — i.e. prediction of the content of an attribute above a cut-off based on these estimates is quite different to that based on true block values. Assessment of project economics (or other critical decision making) based on such distorted grade-tonnage curves will be riskier than necessary. While estimation of very large blocks, say similar in dimensions to the sampling grid, will result in lower estimation variance, it also implies very low selectivity, which is often an unrealistic assumption. This paper presents an overview of the geostatistical approach to solving this problem: non-linear estimation. Linear estimation is compared to non-linear estimation, the motivations of non-linear approaches are presented. A summary of the main geostatistical non-linear estimators is included. In a non-linear estimation we estimate, for each large block (by convention called a "panel") the proportion of SMU-sized parcels above a cut-off grade or attribute threshold. A series of proportions above cut-off defines the SMU distribution. Use of such non-linear estimates reduces distortion of grade-tonnage curves and allows for better decision making. A partial bibliography of key references on this subject is included.

***Key Words**: geostatistics, non-linear estimation, mining, environmental contamination, grade-tonnage curve, indicator, Gaussian transformation, lognormal distribution*

Introduction

This paper uses mining terminology. This is convenient because: (1) this is the language of most of the geostatistical literature (even environmental and petroleum papers refer to grade tonnage relationships, nugget effects, selective mining units and so on) and (2) the main audience for this paper is likely to have a mining background. We hope non-miners are tolerant of this bias.

This is a review paper and as such, an extensive bibliography is given. This bibliography goes beyond the papers cited in the text of our paper and is intended to give readers access to the primary literature.

Linear methods

First things first — what is a linear interpolator?

Inverse Distance Weighting (IDW) interpolators are linear, as is Ordinary Kriging (OK). What do we mean by the term "linear interpolator"?

A relatively non-mathematical understanding of linear weighted averaging can be gained from thinking about linear regression. In linear regression, the relationship between two variables, x and y, is considered to be a straight line (i.e. *linear*). The formula for this straight line is simple:

$$y = ax + b$$

Where a is the slope of the line and b is the value of y when x equals zero (i.e. the y-intercept). If we specify a particular value of x we can therefore conveniently determine the expected y value corresponding to this x. It doesn't matter whether we specify an x value which is very small or very large, or anywhere in-between: the relationship between x and y is always the same — the specified straight line. In other words, the formula used to estimate y does *not* alter as the magnitude of the x value changes.

A linear interpolator has this property: the weights we assign to each of the N sample locations inside our estimation neighbourhood are *independent of the specific data values at these locations*. Think about the simplest kind of linear interpolator, IDW. An IDW estimate assigns the weight to a sample located within the estimation neighbourhood as:

$$\lambda_i = \frac{\frac{1}{d_i^{\alpha}}}{\sum_{j=1}^{N} \frac{1}{d_j^{\alpha}}}$$

Where λ are the weights, d are the distances from each sample location to the centroid of the block to be estimated and α is the power[1]. Once the power to be used is specified, the i^{th} sample is assigned a weight that depends solely upon its location (distance d_i to the centroid). Whether the sample at this location had an average or extreme value does not have any impact whatsoever on the assignment of λ.

OK is a more sophisticated linear interpolator proposed by Matheron (1962, 1963a, 1963b). OK's advantage over IDW as a linear estimator is that it ensures minimum estimation variance given:

(1) A specified model spatial variability (i.e. variogram or other characterisation of spatial covariance/correlation), and

(2) A specified data/block configuration (in other words, the "geometry" of the problem).

The second criterion involves knowing the block dimensions and geometry, the location and support of the informing samples, and the search (or "Kriging neighbourhood") employed. Minimum estimation variance simply means that the estimation error is minimised by OK. Given an appropriate variogram model, OK will outperform IDW because the estimate will be smoothed in a manner conditioned by the spatial variability of the data (known from the variogram).

Now, contrast linear regression with *non-linear* regression. There are many types of non-linear relationships we can imagine between *x* and *y*, a simple example being:

$$y = ax^2 + b$$

This is a quadratic (or parabolic) regression, available in most modern spreadsheet software, for example. Note that the relationship between *x* and *y* is now clearly *non-linear* the nature of the relationship between *x* and *y* is clearly dependent upon the particular *x* value considered. Non-linear geostatistical estimators therefore allocate weights to samples that are functions of the grades themselves and not solely dependent on the location of data.

Non-linear interpolators

Limitations of linear Interpolators

The fundamental limitations of linear estimation (of which OK provides the best solution) are straightforward:

[1] The denominator of this fraction expresses the weight calculated as a proportion of the *total* weight allocated to all samples found within the search

1. We may be motivated to estimate the distribution rather than simply an expected value at some location (or over some area/volume, if we are talking about block estimation). Linear estimators cannot do this. The cases abound: recoverable ore reserves in a mine, the proportion of an area exceeding some threshold of contaminant content in an environmental mapping, etc.

2. We are dealing with a strongly skewed distribution, eg. a precious metal or uranium deposit, and simply estimating the mean by a linear estimator (for example by OK) is risky, the presence of extreme values making any linear estimate very unstable. We may require a knowledge of the distribution of grades in order to get a better estimate of the mean. This usually involves making assumptions about the distribution (for example, what is the shape of the tail of the distribution?) even in situations where we are ostensibly "distribution free" (for example using IK).

3. We may be studying a situation where the arithmetic mean (and therefore the linear estimator used to obtain it) is an inappropriate measure of the average, for example in situations of non-additivity like permeability for petroleum applications or soil strength for geological engineering applications.

The specific problem of estimating recoverable resources was the origin of non-linear estimation and has been the main application.

From a geostatistical viewpoint, non-linear interpolation is an attempt to estimate the *conditional expectation*, and further the *conditional distribution* of grade at a location, as opposed to simply predicting the grade itself. In such a case we wish to estimate the mean grade (*expectation)* at some location under the condition that we know certain nearby sample values (*conditional expectation).* This conditional expectation, with a few special exceptions (eg. under the Gaussian Model – see later) is *non-linear.*

In summary, non-linear geostatistical estimators are those that "… use non-linear functions of the data to obtain (or approximate) …" the conditional expectation. Obtaining this conditional expectation is possible, in particular through the probability distribution:

$$\Pr\left[Z(x_o)|Z(x_i)\right]$$

> This reads: "… the probability of the grade at location x_o given the known sampling information at locations $Z(x_i)$ (i.e. $Z(x_1)$, $Z(x_2)$ ….$Z(x_N)$." This is the *conditional distribution* of grade at that location. Once we know (or approximate) this distribution, we can predict grade tonnage relationships (eg. "how much of this block is above a cut-off Z_C ?).

Available methods

There are many methods now available to make local (panel by panel) estimates of such distributions, some of which are:

- Disjunctive Kriging – DK – (Matheron, 1976, Armstrong and Matheron, 1986a, 1986b);
- Indicator Kriging – IK – (Journel, 1982, 1988) and variants (Multiple Indicator Kriging; Median Indicator Kriging, etc.);
- Probability Kriging – PK – (Verly and Sullivan, 1985);
- Lognormal Kriging – LK – (Dowd, 1982);
- Multigaussian Kriging – MK – (Verly and Sullivan, 1985, Schofield, 1989a, 1989b);
- Uniform Conditioning – UC – (Rivoirard, 1994, Humphreys, 1998);
- Residual Indicator Kriging – RIK – (Rivoirard, 1989).

In a non-linear estimation we estimate, for each large block (by convention called a "panel") the proportion of SMU-sized parcels above a cut-off grade or attribute threshold. A series of proportions above cut-off defines the SMU distribution.

Note that there is a very long literature warning strongly against estimation of small blocks by linear methods (Armstrong and Champigny, 1989; David, 1972; David 1988; Journel, 1980, 1983, 1985; Journel and Huijbregts, 1978; Krige, 1994, 1996a, 1996b, 1997; Matheron, 1976, 1984; Ravenscroft and Armstrong, 1990; Rivoirard, 1994; Royle, 1979). By "small blocks", we mean blocks that are considerably smaller than the average drilling grid (say appreciably less than half the size, although in higher nugget situations, blocks with dimensions of half the drill spacing may be very risky).

The authors strongly reiterate this warning here. The prevalence in Australia of estimating blocks that are far too small is symptomatic of misunderstanding of basic geostatistics. Even estimating such small blocks directly by a non-linear estimator may be incorrect and risky. When using non-linear estimation for recoverable resources estimation in a mine, the panels should generally have dimensions approximately equal to the drill spacing, and only in rare circumstances (i.e. strong continuity) can significantly smaller panels be specified.

Non-linear estimation provides the solution to the "small block" problem. We *cannot* precisely estimate small (SMU-sized) blocks by direct linear estimation. However, we *can* estimate the proportion of SMU-sized blocks above a specified cut-off, within a

panel. Thus, the concept of change of support is critical in most practical applications of non-linear estimation.

Support effect

Definition

"Support" is a term used in geostatistics to denote the volume upon which average values may be computed or measured. Complete specification of support includes the shape, size and orientation of the volume. If the support of a sample is very small in relation to other supports considered, eg. drill hole sample upon which a gold assay has been made, it is sometimes assumed to correspond to "point support".

Grades of mineralisation measured on a small support (eg. drill hole samples) can be much richer or poorer than grades measured on larger supports, say selective mining units (SMU) blocks. The grades on smaller supports are said to be more dispersed than grades on larger supports. Dispersion is usually measured by variance.

Although the global mean grades measured (or estimated) on different supports (at zero cut-off) are the same, the variance of the smaller supports is higher, i.e. very high drill hole sample grades are possible, but large mining blocks have a smoother distribution of grades (fewer very high and very low grades). "Support effect" is this influence of the support on the distribution of grades.

The necessity for change of support

Change of support is vital for predicting recoverable reserves if we intend to selectively mine a deposit. Before committing the capital required to mine such a deposit, an economic decision must be made based only on the samples available from exploration drilling. Because mining does *not* proceed with a selection unit of comparable size to the samples, the difference in support between the samples and the proposed SMU must be accounted for in any estimate to obtain achievable results. When there is a large nugget effect, or an important short-scale structure apparent from the variography, then the impact of change of support will be pronounced.

The histogram of drill hole samples will usually have a much longer "tail" than the histogram of mining blocks. Simplistic variance corrections, for example affine corrections, do not reflect the fact that, in addition to variance reduction, change of

support also involves symmetrisation of the histogram[2]. This is especially important in cases where the histogram of samples is highly skewed.

Recoverable resources

Recoverable resources are the portion of in-situ resources that are recovered during mining. The concept of recoverable resources involves both technical considerations, such as cut-off grade, SMU definition, machinery selection etc., and also economic/financial considerations such as site operating costs, commodity prices outlook, etc. In this paper, only technical factors are considered. Recoverable resources can be categorised as either global or local recoverable resources. Global recoverable resources are estimated for the whole field of interest; eg. estimation of recoverable resources for the entire orebody (or a large well-defined subset of the orebody like an entire bench)[3]. Local recoverable resources are estimated for a local subset of the orebody; eg. estimation of recoverable resources for a 25m x 50m x 5m panel.

A summary of main non-linear methods

Indicators

The use of indicators is a strategy for performing structural analysis with a view to characterising the spatial distribution of grades at different cut-offs. The transformed distribution is binary, and so by definition does not contain extreme values. Furthermore, the indicator variogram for a specified cut-off z_c is physically interpretable as characterising the spatial continuity of samples with grades exceeding z_c. Indicator transformations may thus be conceptually viewed as a digital contouring of the data. They give very valuable information on the geometry of the mineralisation.

A good survey of the indicator approach can be found in the papers of Andre Journel (eg. 1983, 1987, 1989).

An indicator random variable $I(x,z_c)$ is defined, at a location x, for the cut-off z_c as the binary or step function that assumes the value 0 or 1 under the following conditions:

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[2] This symmetry can be demonstrated via the central limit theorem of classical statistics, which states that the means of repeated samplings of any distribution will have a distribution which is normal, regardless of the underlying distribution.. When we consider block support, the aggregation of points to form blocks will thus deskew the histogram. In the ultimate case, we have a single "block", being the entire zone of stationarity and there is *no* skewness as such.

[3] Global recoverable resources can be very useful as checks on local recoverable estimation, a good first pass valuation or can be used for checking the impact on grade-tonnage relationships of changing SMU, bench-height studies, etc. They are not specifically discussed in this paper. The interested reader is referred to Vann and Sans (1995).

$$I(x,z_c) = 0 \qquad \text{if } Z(x) \le z_c$$
$$I(x,z_c) = 1 \qquad \text{if } Z(x) > z_c$$

The indicators thus form a binomial distribution, and we know the mean and variance of this distribution from classical statistics:

$$m = p$$
$$\sigma^2 = p(1-p)$$

> Where p is the proportion of 1's as defined above (for example, if the cut-off, z_c is equal to the median of the grade distribution, p takes a value of 0.5, and the maximum variance is defined as 0.25).

After transforming the data, indicator variograms can be calculated easily by any program written to calculate an experimental variogram. An indicator variogram is simply the variogram of the indicator.

Indicator Kriging

Indicator Kriging is kriging of indicator transformed values using the appropriate indicator variogram as the structural function. In general the kriging employed is ordinary kriging. (OK). An IK estimate (i.e. kriging of a single indicator) must always lie in the interval [0,1], and can be interpreted either as

1. probabilities (the probability that the grade is above the specified indicator) or
2. as proportions (the proportion of the block above the specified cut-off *on data support*).

In addition to its uses for indicator kriging (IK), multiple indicator kriging (MIK), probability kriging (PK) and allied techniques, the indicator variogram can be useful when making structural analysis to determine the average dimensions of mineralised pods at different cut-offs. Indicators are also useful for charactering the spatial variability of categorical variables (eg. presence or absence of a specific lithology, alteration, vein type, soil type, etc.). Henstridge (1998) presents examples of such applications for an iron deposit and Gossage (1998) give a more general overview of such applications of indicator kriging.

Multiple Indicator Kriging

Multiple indicator kriging (MIK) involves kriging of indicators at several cut-offs (see various publications by Andre Journel in the references to this paper as well as Hohn, 1988 and Cressie, 1993). MIK is an approach to recoverable resources estimation which is robust to extreme values and is practical to implement. Theoretically, MIK gives a worse approximation of the conditional expectation than disjunctive kriging

(DK), which can be shown to approximate a full co-kriging of the indicators at all cut-offs, but does not have the strict stationarity restriction of DK.

The major difficulties with MIK can be summarised as:

1. Order relation problems: i.e. because indicator variogram models may be inconsistent from one cut-off to another we may estimate more recovered metal above a cut-off z_{c2} than for a lower cut-off z_{c1}, where $z_{c1} < z_{c2}$, which is clearly impossible in nature. While there is much emphasis on the triviality of order relation problems and the ease of their correction in the literature, the authors have observed quite severe difficulties in this regard with MIK. The theoretical solution is to account for the cross-correlation of indicators at different cut-offs in the estimation by co-kriging of indicators, but this is completely impractical from a computational and time point of view. In fact, the motivation for developing probability kriging (PK) was to approximate full indicator co-kriging (see below).

2. Change of support is not inherent in the method. In the authors' experience, most practical applications of MIK involve using the affine correction, which assumes that the shape of the distribution of SMU's is identical to that of samples, the sole change in the distribution being variance reduction as predicted by Krige's Relationship. There are clear warnings in the literature (by Journel, Isaaks and Srivastava, Vann and Sans, and others) about the inherent *deskewing* of the distribution when going from samples to blocks. The affine correction is not suited to situations where there is a large decrease in variance (i.e. where the nugget is high and/or there is a pronounced short-scale structure in the variogram of grades). Other approaches can be utilised, e.g. lognormal corrections (very distribution dependent), or conditional simulation approaches (costly in time). A new proposal for change of support in MIK is given by Khosrowshahi *et al*. (1998).

Median Indicator Kriging

Median indicator kriging is an approximation of MIK which assumes that the spatial continuity of indicators at various cut-offs can be approximated by a single structural function, that for $z_c = \tilde{m}$, where $\tilde{m}$ is the median of the grade distribution. The indicator variogram at (or close to) the median is sometimes considered to be "representative" of the indicator variograms at other cut-offs. This may or may not be true, and needs to be checked. The clear advantage of median indicator kriging over MIK is one of time (both variogram modelling and estimation). The critical risk is in the adequacy of the implied approximation. "If there are noticeable differences in the shape of indicator variograms at various cut-offs, one should be cautious about using median indicator kriging" (Isaaks and Srivastava, 1989, pp 444). Hill et al (1998) and Keogh and Moulton (1998) present applications of the method.

Probability Kriging

Probability kriging (PK) was introduced by Sullivan (1984) and a case study is given in Verly and Sullivan (1985). It represents an attempt to alleviate the order relationship problems associated with MIK, by considering the data themselves (actually their standardised rank transforms, distributed in [0,1]) in addition to the indicator values. Thus a PK estimate is a co-kriging between the indicator and the rank transform of the data *U*. When performed for *n* cut-offs, it requires the modelling of *2n+1* variograms: *n* indicator variograms, *n* cross-variograms between indicators and *U*, and finally the variogram of *U*. The hybrid nature of this estimate as well as the time-consuming complexity of the structural analysis makes it rather unpractical.

Indicator Co-kriging and Disjunctive Kriging

In general, any practical function of the data can be expressed as a linear combination of indicators:

$$f(Z) = \sum_n f_n I(Z, z_n)$$

Thus, estimating $f(Z)$ amounts to estimating the various indicators. The best linear estimate of these indicators is their full co-kriging, which takes into account the existing correlations between indicators at various cut-offs. Full indicator co-kriging (also called Disjunctive Kriging, abbreviated to DK) theoretically ensures consistency of the estimates (reducing order relationships to a minimum or eliminating them altogether): this makes the technique very appealing, but there is a heavy price to pay: if *n* indicators are used, n^2 variograms and cross-variograms need to be modelled, and this is unpractical as soon as *n* gets over 5 or 6, even with the use of modern automatic variogram modelling software.

The various non-linear estimation methods can be considered as ways of simplifying the full indicator cokriging. Roughly speaking, there are three possible paths to follow (Rivoirard, 1994):

1. Ignore the correlations between indicators: this is the choice made by MIK already discussed. The authors consider this a fairly drastic choice.

2. Assume that there is *intrinsic correlation*, i.e.that all variograms and cross-variograms are multiples of one unique variogram. In that case, cokriging is strictly equivalent to kriging; this is the hypothesis underlying median IK. Needless to say, unfortunately, this very convenient assumption is rarely true in practice (see median IK, above).

3. Express the indicators as linear combinations of uncorrelated functions (*orthogonal functions)*, which can be calculated from the data. Cokriging of the indicators is then equivalent to separate kriging of the orthogonal functions; this

decomposition of the indicators is the basis of residual indicator kriging (RIK) and of isofactorial disjunctive kriging.

Residual Indicator Kriging

In this particular model, within the envelope defined by a low cut-off, the higher grades are randomly distributed. The proximity to the border of the envelope has no direct incidence on the grade, and this corresponds to some types of vein mineralisation, where there is little correlation between the geometry of the vein and the grades. The validity of the model is tested by calculating the ratios

$$\frac{\gamma_{ij}(h)}{\gamma_i(h)}$$

(cross-variograms of indicators over variograms of indicator) for the cut-offs z_j higher than z_i. If these ratios remain approximately constant, then the model is appropriate. Note that an alternative "decreasing" model exists where one compares the cross-variograms to the variogram associated with the highest cut - off (instead of the lowest).

The residuals are defined from the indicator functions by $H_i(x) = \frac{I(Z(x), z_i)}{T_i} - \frac{I(Z(x), z_{i-1})}{T_{i-1}}$ where $T_i = E\left[I(Z(x), z_i)\right]$, i.e. the proportion of grades higher than the cut-off z_i.

i.e.

$$H_0(x) = 1$$

$$H_1(x) = \frac{I(Z(x), z_1)}{T_1} - 1$$

.....

$$H_n(x) = \frac{I(Z(x), z_n)}{T_n} - \frac{I(Z(x), z_{n-1})}{T_{n-1}}$$

The $H_i(x)$ are uncorrelated and we have:

$$\frac{I(Z(x), z_i)}{T_i} = \sum_{j=0}^{i} H_j(x)$$

This means that the indicators can be factorised. In order to get a disjunctive estimate of $f(Z(x))$, it is enough to krige separately each of the residuals $H_i(x)$. The T_i are simply estimated by the means of the indicators, $I(Z(x), z_i)$.

In practice, the residuals are calculated at each data point, their variograms are then evaluated and independent krigings are performed. Another check of the model consists in directly looking at the cross variograms of the residuals: if they are flat, indicating no spatial correlation, the model works. Thus, essentially this model requires no more calculations than indicator kriging, while being more consistent when it is valid.

The reader is referred to Rivoirard (1994, chapter 4) for a fuller explanation and a case study (chapter 13) of this approach.

Residual indicators is one way to co-krige indicators by separately kriging independent combinations of them and recombining these to form the co-kriged estimate. Like MIK, this method involves working with many indicators and the same number of variograms. Thus, it can be time consuming.

Isofactorial Disjunctive Kriging

There are several versions of isofactorial DK, by far the most common is Gaussian DK.

Gaussian DK is based on an underlying "diffusion" model (where, in general, grade tends to move from lower to higher values and vice versa in a relatively continuous way).

The initial data are transformed into values with a Gaussian distribution, which can easily be factorised into independent factors called Hermite polynomials (see Rivoirard, 1994 for a full explanation and definition of Hermite polynomials and disjunctive kriging). In fact, any function of a Gaussian variable, including indicators, can be factorised into Hermite polynomials. These factors are then kriged separately and recombined to form the DK estimate. The major advantage of DK is that you only need to know the variogram of the Gaussian transformed values in order to perform all the krigings required. The basic hypothesis made is that the bivariate distribution of the transformed values is bigaussian, which is testable. Although order relationships can occur, they are very small and quite rare in general. A very powerful and consistent change of support model exists for DK: the discrete Gaussian model (see Vann and Sans, 1995).

Gaussian disjunctive kriging has proved to be relatively sensitive to stationarity decisions, (in most cases simple kriging is used in the estimation of the polynomials). DK should thus only be applied to *strictly* homogeneous zones.

Uniform Conditioning

Uniform conditioning (UC) is a variation of Gaussian DK more adapted to situations where the stationarity is not very good.

In order to ensure that the estimation is locally well constrained, a preliminary ordinary kriging of relatively large panels is made, and the proportions of ore per panel are conditional to that kriging value.

UC is a relatively robust technique. However, it does depend heavily upon the quality of the kriging of the panels. As for DK, the discrete Gaussian model ensures consistent change of support. Humphreys (1998) gives a case study of application of UC to a gold deposit.

Lognormal Kriging

Lognormal kriging (LK) is not linked to an indicator approach and belongs to the conditional expectation estimates.

If the data are truly lognormal, then it is possible, by taking the log, and assuming that the resulting values are multigaussian, to perform a lognormal kriging. The resulting estimate is the conditional expectation and is thus in theory the best possible estimate. This type of estimation has been used very successfully in South Africa. Unfortunately the lognormal hypothesis is very strict: any departure can result in completely biased estimates.

Multigaussian kriging

A generalisation of the lognormal transformation is the Gaussian transformation which applies to any reasonable initial distribution. Again, under the multigaussian hypothesis, the resulting estimate represents the conditional expectation and is thus optimal. This is a very powerful estimate much more largely applicable than lognormal kriging, but requires very good stationarity to be used with confidence. Compared to Gaussian DK, it is completely consistent, but based on stronger multigaussian assumptions and its application to block estimation is more complex.

Conclusions and recommendations

1. As we approach the end of this century, and nearly 40 years since Matheron's pioneering formulation of the Theory of Regionalised Variables, there are a large number of operational non-linear estimators to choose from. Understanding the underlying assumptions and mathematics of these methods is critical to making informed choices when selecting a technique.

2. We join the tedious chorus of geostatisticians over many years and recommend that linear estimation of small blocks be consigned to the past, unless it can be explicitly proved through very simple and long known kriging tests that such estimation is adequate. It is our professional responsibility to change to culture of "providing what is asked for" regardless of the demonstrable and potentially serious financial risks of such approaches.

"Small block" OK or IDW estimates should no longer be acceptable as inputs to important financial decisions.

3. A particular non-linear method is often applied by a given practitioner without considering, for the data set in hand, whether the main assumptions of that method are realistic. Some of these assumptions may be testable, for example assumptions about the cross correlations of indicators or assumptions about the nature of "edge effects". Testing of such assumptions is rarely performed, in our experience. We therefore recommend that such tests be implemented (see Rivoirard, 1994).

4. The issue of change of support is critical in estimation of recoverable resources, and as such should remain a major topic in our field. The major criticisms of MIK, the most widely applied non-linear estimation method in Australia, have centred on change of support (as pointed out by Glacken and Blackney, 1998). *The whole problem of recoverable reserves is the problem of change of support.* We recommend that practitioners become highly familiar with the issue of change of support and bring a sophisticated appreciation of this problem to their practice.

5. Conditional simulation (another non-linear method) is now within the abilities of inexpensive desktop computing, leading to another possible future route to recoverable resources. For a given block, the average of a set of *n* conditional simulations *is exactly equal to the kriged estimate of that block* In fact, as the full conditional distribution of the block grade is accessible, any non linear estimate can be calculated. Now, it must be clear that simulations are based exactly on the same type of hypotheses as most non-linear estimation methods (stationarity, representativity of the variogram, etc.) and, from this viewpoint, they need to be assessed as critically as any estimation method. While multiple conditional simulation is an inefficient route to local recoverable resources in 1998, the exponential increase in computational speed witnessed in the past decade suggests that it will soon become viable.

6. We hope that geostatistics does not go down a "proprietary" route. By this we mean that the *algorithmic* basis of geostatistical methods should rightly be in the public domain (and thus debatable and open to cross-validation). The publication of GSLIB (Journel and Deutsch, 1998) sets standards in this regard. Publication of actual source codes is more debatable of course, and there is certainly no consensus about it.

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