Moving forward from traditional optimization: grade uncertainty and risk effects in open-pit design

R. Dimitrakopoulos, C. T. Farrelly and M. Godoy

Synopsis

An economic argument is presented for the incorporation of quantitative modelling of the uncertainty of grade, tonnage and geology into open-pit design and planning. Two new implementations of conditional simulation—the generalized sequential Gaussian simulation and direct block simulation—are outlined. An optimization study of a typical disseminated, low-grade, epithermal, quartz breccia-type gold deposit is used to highlight the differences between the financial projections that may be obtained from a single orebody model and the range of outcomes produced when, for example, fifty deposit simulations are run. The effects on expectations of net present value, production cost per ounce, mill feed grade and ore tonnage are presented as examples and periods with a high risk of negative discounted cash flow are identified. Further integration of uncertainty into optimization algorithms will be needed to increase their efficacy.

The quantification of uncertainty and risk has major implications for open-pit design and production scheduling as it relates to the management of cash flows in the order of millions of dollars. Optimization in mine planning has been accepted as a set of techniques that introduce analytical mathematical methods into planning.[1] The most common approach in open-pit design and planning is based on the Lerchs–Grossmann three-dimensional graph theory, implemented in industry applications as the nested Lerchs–Grossmann algorithm.[2,3,4] A key concern when dealing with risk and uncertainty, particularly considering the financial implications of decisions made on the basis of optimization studies, may be expressed by the statement: ‘I would rather be approximately right than precisely wrong’. This statement hints at a way to address the uncertainty present in any mine design and plan. To deal with, manage and benefit from risk requires further development of quantitative methods used in planning that can minimize the chances of a single, precisely wrong expectation. As a result, strategic investments can be sheltered and operations perform closer to their potential.

Traditional evaluation of mining projects includes drilling and sampling, generating a representative orebody model, deciding mining and processing methods, assessing capital and operating costs and developing a technical and financial life-of-mine plan. In addition, to assess the worth of a project, summary indicators, which include total project size, capital requirements and net present value, are developed and used to generate best decision-making options that result in maximum expected project utility.[5] Although complex in practice, this evaluation process can be seen as a combination of management strategy with a critical understanding and assessment of uncertainty and risk from technical, financial and environmental sources. A critical source of technical risk is in the expected ore grade and tonnage. The ability to model and integrate this risk into optimization and planning is of paramount importance and allows a more informed approach to be taken to the valuation of an asset or design and management of a project.

The presence of geological risk in mining projects is well known and appreciated. During the past few years evolving technologies have allowed direct modelling of geological risk. As a result, several issues have been raised, including the integration of grade uncertainty into pit optimization and recoverable reserves,[6,7] technologies and algorithms for geological risk modelling in pit optimization and production scheduling,[8] risk in mineral projects and ultimate pit limits,[9] impact of high-risk grade zones in optimal pit limits[10] and risk analysis for production scheduling.[11,12] Despite these developments there is an increasing need for further understanding of the main limitations of the traditional, non-risk-based, open-pit optimization approaches and their potential effects on project decision-making.

In the present contribution consideration is given, first, to key reasons why there are benefits from understanding and quantifying uncertainty and risk. A new, fast and efficient conditional simulation framework for the modelling of geological uncertainty in an industrial environment is then outlined. Examples of the reasons why one is interested in quantifying risk in optimization studies are provided, using a study from a typical Australian low-grade gold deposit. Finally, potential future needs are discussed.

Key reasons to undertake quantitative modelling of geological uncertainty and risk

Quantification of geological uncertainty and risk can enhance mining project development and mining operations substantially. Modern project valuation frameworks can elucidate the paramount positive economic effects of the quantification of uncertainty and risk. One such is the ‘real options’ framework,[13,14] a key characteristic of which is the ability to integrate and manage uncertainty and risk and thus enable the sheltering of strategic investments and exposure of their upside potential. In simple terms, real options may be described as the ability to assess the value of starting a project that gives the right, but not the obligation, to commence operations at a cost of, say, $7 000 000 six months from now and/or to assess the value of delaying production to obtain additional information to reduce uncertainty or to quantify the value of building in the flexibility to manage uncertainty and risk at any level or aspect of an exploration or mining venture. Fig. 1 summarizes a comparison between a traditional valuation method that does not account for uncertainty

Paper originally presented at the Strategic mine planning conference held by the Australasian Institute of Mining and Metallurgy in Perth, Western Australia, from 26 to 29 March, 2001; revised manuscript received by the Institution of Mining and Metallurgy on 21 April, 2002. Paper published in *Trans. Instn Min. Metall.* (*Sect. A: Min. tech - nol.*), 111, January–April 2002.

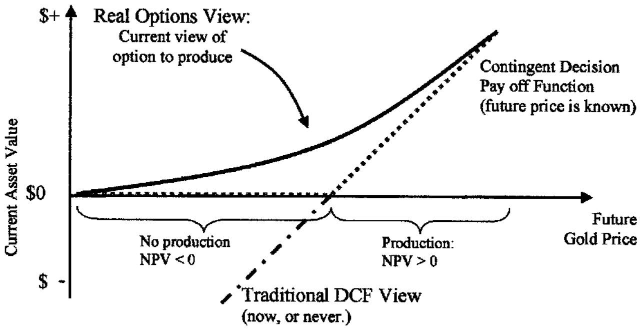


Fig. 1 Accounting for uncertainty increases asset value: example using 'real options' (*solid black line*) whereby possible future changes (uncertainty) are accounted for, versus traditional (deterministic) DCF analysis (*broken grey line)*

and risk, discounted cash flow (DCF) analysis and real options in assessing current asset value. The figure shows an increase in asset value from the simple step of explicitly quantifying uncertainty and integrating the data into financial analysis and decision-making.

The need to quantify uncertainty in asset valuation and decision-making translates to the need to quantify uncertainty and risk in any pertinent components of open-pit design and long-term planning. Project risk may arise from three main sources—technical (geological and mining), financial and environmental. The major source of technical risk is uncertainty in grades, tonnage, geology and geomechanics. Geological risk is seen as the major contributor to not meeting project expectations. For example, at the early stages of a project, when establishing investor confidence and repayment of development capital are vital, Vallee[15] noted that '... in the first year of operation after start-up, 60% of mines surveyed had an average rate of production less than 70% of designed capacity'. Although shortfalls in production are also due to problems in scale-up from pilot-plant to commercial plant, the quantity and grade of ore are a major contributor to potential shortfalls. Shortfalls from mine production predictions are also common in later stages of production and are attributed substantially to geological causes.[16]

For any open-pit design uncertainty over grades, tonnages or geology can be readily modelled and integrated into the optimization and design process so as to provide accurate modelling and quantification of uncertainty and risk, rather than a single estimate assessment, for any pertinent parameter—including the project NPV, expected cash flows, recoverable quantity of mineral and expected production costs. This provides the ability to develop a different, technically sound, risk-based approach to valuing an asset, operation or project as well as quantify, and thus minimize, risk in selection of an appropriate pit design.

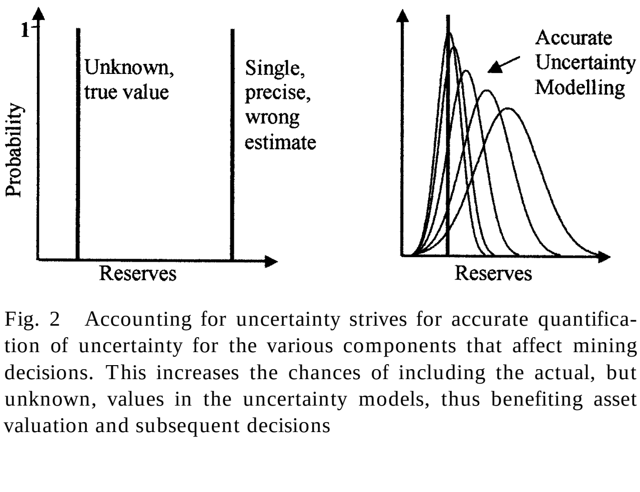


Fig. 2 Accounting for uncertainty strives for accurate quantification of uncertainty for the various components that affect mining decisions. This increases the chances of including the actual, but unknown, values in the uncertainty models, thus benefiting asset valuation and subsequent decisions

Fig. 2 illustrates an assessment of uncertainty for a parameter that may, for example, be the ore reserves in a gold mine. Accounting for uncertainty requires its accurate quantification; thus, technical work and evaluation should stress accurate modelling and quantification of uncertainty and risk, not a single estimate or a qualitative-type assessment. The technical ability to model uncertainty quantitatively with as much accuracy as possible with the information available at a given time is of paramount importance. Conditional simulation technologies offer a first, key step in the modelling of geological risk and are discussed next.

Models of grade uncertainty for the industrial environment

Conditional simulation (CS) is a Monte Carlo-type simulation approach[17] developed for modelling uncertainty in spatially distributed attributes, such as pertinent characteristics of mineral deposits. The idea is to generate equally probable realizations (representations) of the *in-situ* orebody grade and material type variability. All realizations of the orebody are based on and reproduce the available data, their distribution and spatial continuity as well as any other information available for the deposit and attribute under consideration. A large collection of conditionally simulated deposits captures the uncertainty about the orebody and attribute of interest. Examples of the approach and comparisons with traditional orebody modelling can be found elsewhere.[18]

A bottleneck for the conditional simulation technologies and their use in industry is computing speed and efficiency. Despite advances in the computing technologies readily available in desktop computers, simulating an orebody represented by several millions of nodes per realization (for example, large orebodies may require up to 10^8 nodes) is not a fast and simple task that can be performed routinely. An additional complication is that realizations are generated at a quasi-point support and are 'reblocked' to the block sizes needed for various tasks, such as pit optimization and production scheduling studies. Two new implementations of conditional simulation are outlined here. Both have been developed to address industry's needs for computational efficiency and applications with finite memory requirements.

The generalized sequential Gaussian simulation, or GSGS,[19] is a general form of the well-known sequential Gaussian simulation, or SGS.[20] GSGS replaces the node-by-node sequential process in SGS with a group of nodes and the simulation is carried out for groups of nodes simultaneously. The method capitalizes on the fact that, in practice, the simulation grid is usually large and dense, which usually leads to overlapping of neighbourhoods among the closest nodes. The theoretical foundations of the approach are based on the group decomposition of the conditional probability distribution of the attribute considered and the equivalence of SGS to the simulation method based on the so-called LU decomposition of the covariance matrix. The GSGS algorithm for a grid of N nodes divided into k groups of nodes proceeds thus: (1) define a path visiting each group of k nodes of the grid; (2) find a neighbourhood for the current group; (3) generate the simulated values of the current group using the LU method; (4) add the simulated values of the current group into the data-set; and (5) loop until all groups, thus grid nodes, are simulated.

Depending on the design of the groups of nodes versus the number of conditioning data in the neighbourhood the method is up to ten times faster than the more traditional SGS point-by-point algorithm. In addition, the GSGS algorithm is less demanding in terms of memory requirements.

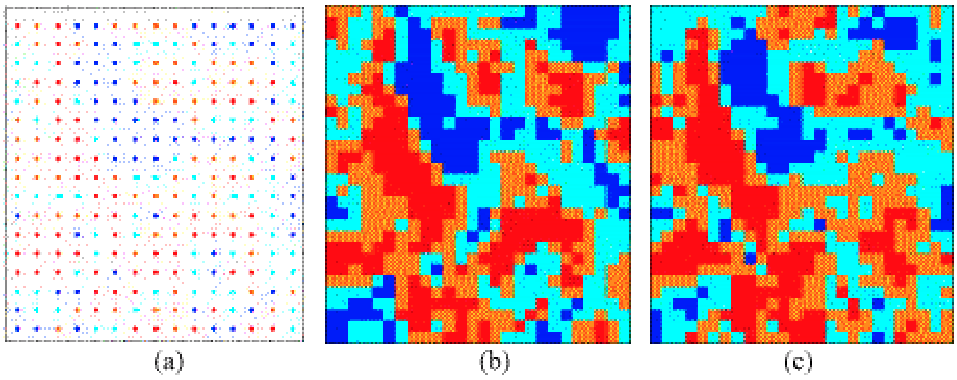


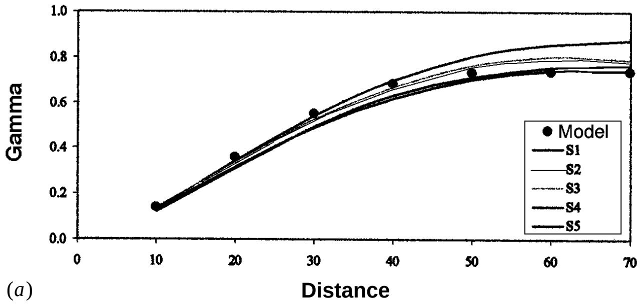
Fig. 3 (*a*) Data-set with 255 samples; (*b*) conditional simulation generated using traditional point-by-point approach; (*c*) realization generated using direct simulation algorithm

Note that if the number of points in the group is one, GSGS is identical to SGS, whereas if all nodes belong to one group, GSGS is identical to the LU simulation method.

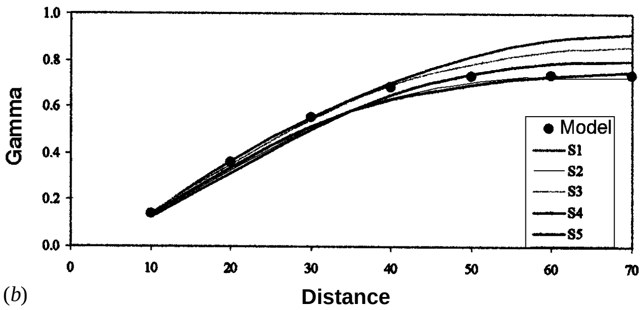
Although substantially more efficient than traditional approaches, the algorithm may still be considerably improved. Consider, for example, a large domain where millions of simulated node values need to be retained as conditional information. The memory requirements alone are huge, not to mention performance losses due to increased search times. To address these issues a direct block support simulation method can be developed on the basis of some of the ideas in the GSGS algorithm. Specifically, a direct block simulation method is developed such that after the simulation of the internal points of each block (group of nodes in GSGS) the simulated block value is calculated and the point values are discarded. The simulated block value is then added to the conditioning data-set. To integrate the block support conditioning data the algorithm is developed in terms of a joint-simulation.[21] The second variable relates to the block values sequentially derived throughout the simulation process. The algorithm provides the means to simulate several hundreds of blocks per second and is substantially faster that any point conditional simulation combined with reblocking.

Fig. 3(*a*) shows a data-set of 255 samples used to simulate 780 block grades. Two images of conditional simulations were generated to permit comparison. The first image is based on point simulation followed by reblocking of 10 × 10 nodes and is shown in Fig. 3(*b*). The second simulation image shown in Fig. 3(*c*) is generated using the direct block simulation approach discussed above, the same block size as previously and 100 points in each block. The direct block simulation reduced the processing time from the traditional point simulation by approximately 2000 nodes per second. Reproduction of the data statistics is not affected and is identical in the two cases. To compare the point-by-point and then reblocking approach with the more efficient direct block simulation Fig. 4 presents the variograms of the two simulation approaches at the block support scale and calculated for five realizations. The comparison with the corresponding variogram model suggests that the two approaches reproduce the spatial continuity of the grades equally well.

The direct block simulation method is substantially faster than the point-by-point simulation, more efficient in terms of computing requirements and reliable in terms of reproduction of the sample statistics. An additional advantage is the ability to simulate models with different block sizes and shapes, which is often required to comply with the geometrical complexity of typical geological domains.



(*a*)



(*b*)

Fig. 4 Variogram reproductions corresponding to (*a*) proposed direct block simulation algorithm (group size 100) and (*b*) traditional point-by-point approach (group size 1)

Avoiding single and possibly precisely wrong options: quantification of geological uncertainty in examples

The availability of conditional simulation technologies allows the integration of geological uncertainty into optimization studies and related decision-making. This differs from the traditional grade-estimation model used in pit optimization studies. The following examples elucidate the differences.

The examples given here are taken from the optimization study of a typical, disseminated, low-grade, epithermal, quartz breccia-type gold deposit, hosted in intermediate–felsic volcanic rocks and sediments. Free milling and refractory ores are to be mined by open-pit methods. Ore is to be processed via a carbon-in-leach processing plant, with a flotation circuit added for the refractory ore. The question of geological uncertainty and risk in the design, planning and production expectations is accentuated by the generally low ore reserve grade and a variable, depressed metal price. The example starts with the traditional way of pit optimization, in which an estimated orebody model of the deposit is used to

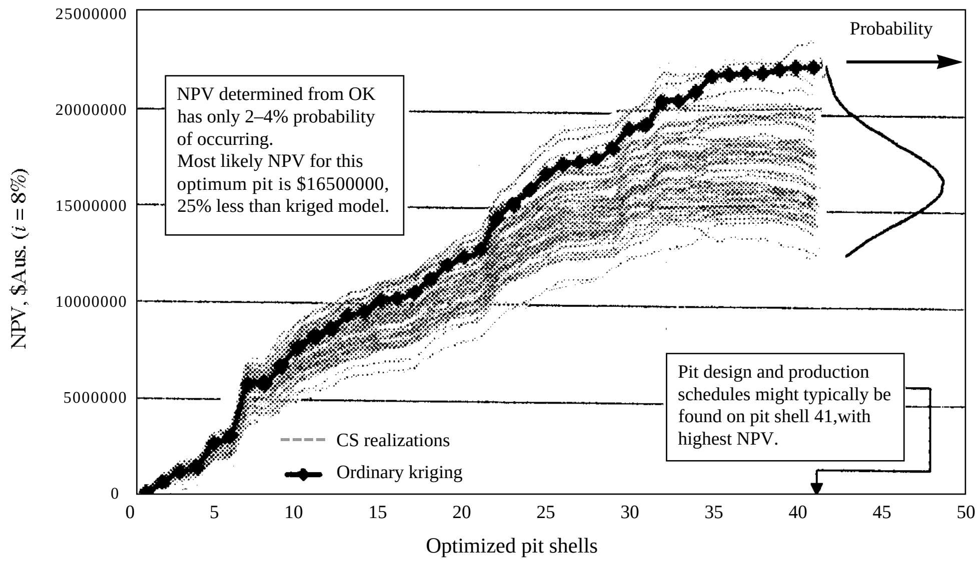


Fig. 5 Geological uncertainty and risk in NPV of disseminated gold deposit

produce a design. Subsequently, 50 realizations of the deposit are developed to quantify geological risk for the given mine design and long-term mine plan. This is implemented by replacing the estimated orebody model with each one of the 50 simulations and rerunning the optimization while the other mining and economic parameters are kept the same.

Fig. 5 shows the analysis of net present value. Orebody simulations have produced a range of financial outcomes,

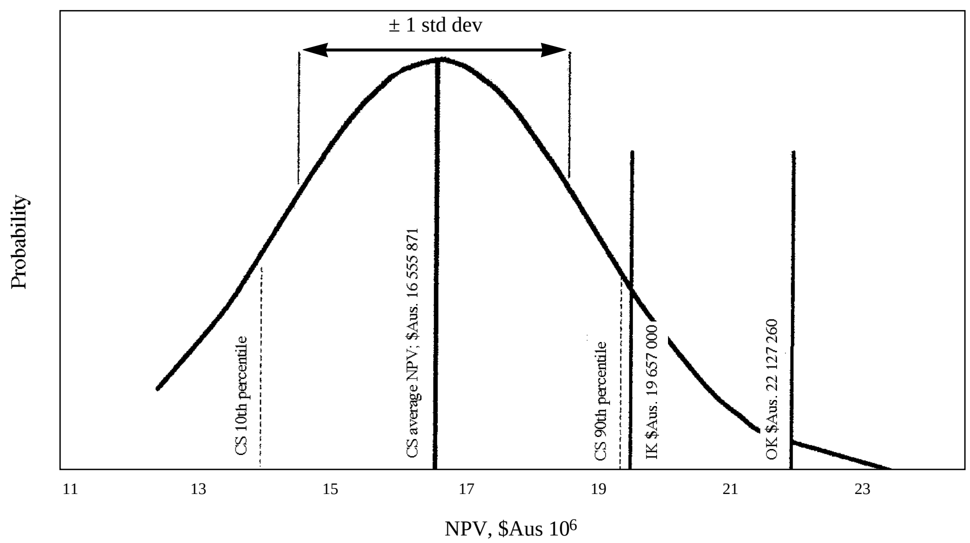


Fig. 6 Distribution of NPV from conditionally simulated realizations and kriged orebody estimates

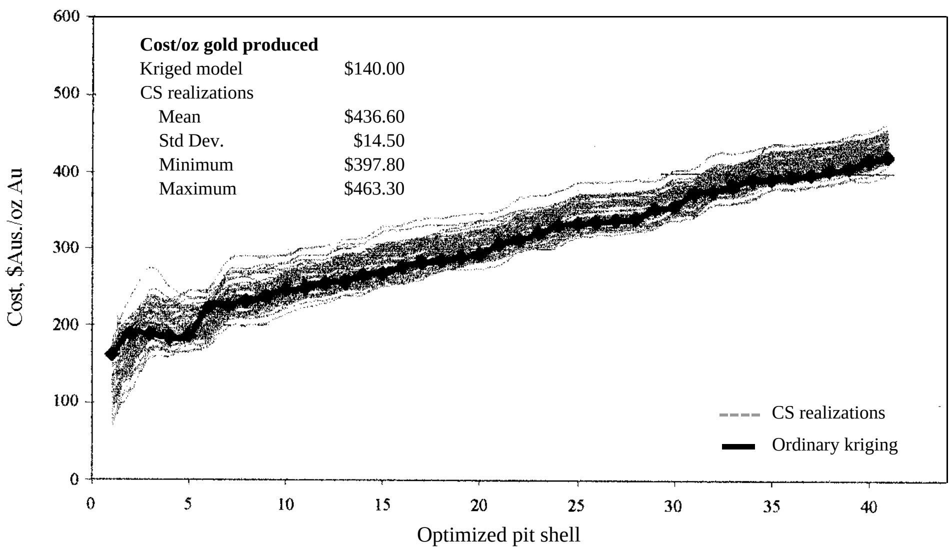


Fig. 7 Geological uncertainty in cost of production per ounce of metal produced

which in this example is in contrast to the single estimate expected from the traditional approach. The project NPV is shown to vary drastically, with about 80% of the outcomes covering a range of $Aus. 5 000 000. The NPV outcome for the traditional approach is shown to be higher than the ninety-fifth quantile of the distribution, i.e. there is a 95% probability of the project returning a lower NPV than predicted by the estimated orebody model. The median NPV from the conditional simulations is $Aus. 16 000 000, approximately 25% lower than the estimated model indicates; and the worst-case scenario from the simulations has an NPV 45% lower than the estimated orebody. Fig. 6 shows the distribution of project NPV more clearly and summarizes the distribution of possible project NPV for the pit design considered.

The operating cost of production may be cited as a benchmark when comparing gold-mining projects. Fig. 7 shows the geological uncertainty and risk integrated into the expected production cost per ounce of gold produced from the deposit considered here and the given mine layout. The analysis in Fig. 7 shows that the cost of production per ounce of gold is most likely to be underestimated from the kriged grade model. However, the small range of outcomes would provide confidence that the cost of production for the project is not likely to exceed $Aus. 463/oz gold. This may be very useful quantitative information to have if the company involved in the project is sensitive to production cost variation. The cost per ounce is shown in Fig. 7 in terms of nested pit shells for comparison with the NPV chart. Note that the range, or spread, of cost per ounce outcomes is fairly constant across all the pit optimization shells. This shows that the cost of production is insensitive to the size of the open-pit and is not significantly improved by increasing the scale of the mining operation. The cost per ounce on a period or annual basis could also be calculated to investigate the cost profile over time.

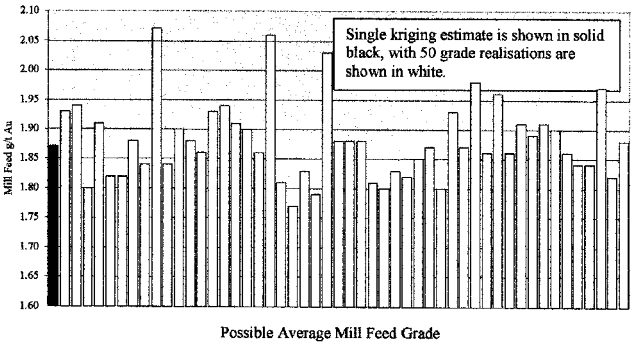


Fig. 8 Average grade of mill feed ore over life of mine for series of orebody realizations and for single estimate

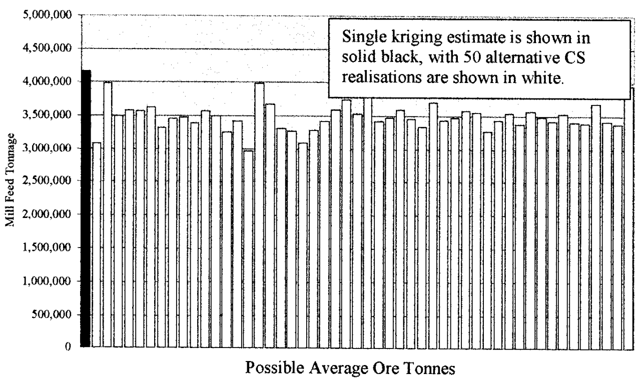


Fig. 9 Average ore tonnages available to mill over life of mine for series of CS realizations and for single kriging estimate

The physical parameters of ore tonnes mined and milled are also major sources of risk, particularly in the early years of a project. The risk of designing and constructing a plant unsuited to the available ore feed will be better understood when the uncertainty in the feed quantity and grade is known. As an example, uncertainty in the mill feed grade to a gold plant was analysed. The analysis in Fig. 8 is the average mill feed grade for the life of the project. The figure shows a large range of possible average feed grades for the project, information useful to have when designing a processing plant.

The conjugate partner to mill feed grade is the ore tonnage, shown in Fig. 9. The simulated realizations show consistently lower total ore tonnage, down by an average of 12.5% compared with the kriged model. The variable grade and lower tonnage of mill feed indicated by the simulated realizations suggest that this project will have difficulty in achieving scheduled mill throughput and feed grade for the life of the mine if based on the kriging model. The lower ore tonnage indicated from the simulations may suggest a design change for the processing plant.

In addition to sensitivity analysis on key 'life of mine' project parameters, geological uncertainty in the mine production schedule can be quantified. Consider, for example, the DCF for a mine calculated for production periods of three months. What variation is expected to arise from a production schedule DCF owing to geological uncertainty in the ore-reserve estimation model? Are there periods of greater

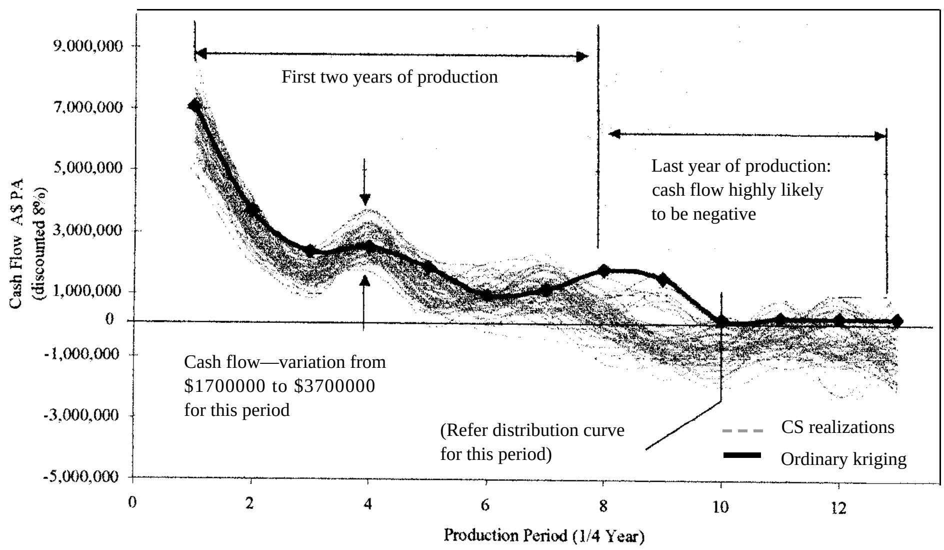


Fig. 10 Distribution of outcomes for discounted cash flow by three-month production periods

risk, and when do such periods occur? Fig. 10 demonstrates the uncertainty in quarterly DCF, in comparison with the single estimate used in Figs. 8 and 9. Fig. 10 shows that the anticipated cash flow for this minable reserve has a reasonable probability of materializing for accounting periods during the first two years of this project. It is more likely that cash flows will be less than forecast, but there is a small probability that cash flows would actually exceed expectations during the first two years. The probability of the last year of production achieving the forecast cash flow is very low; it is much more likely that the DCF during the last year (periods 8–12) will be negative. The cash-flow distribution in period 10 is shown as a probability density function in Fig. 11.

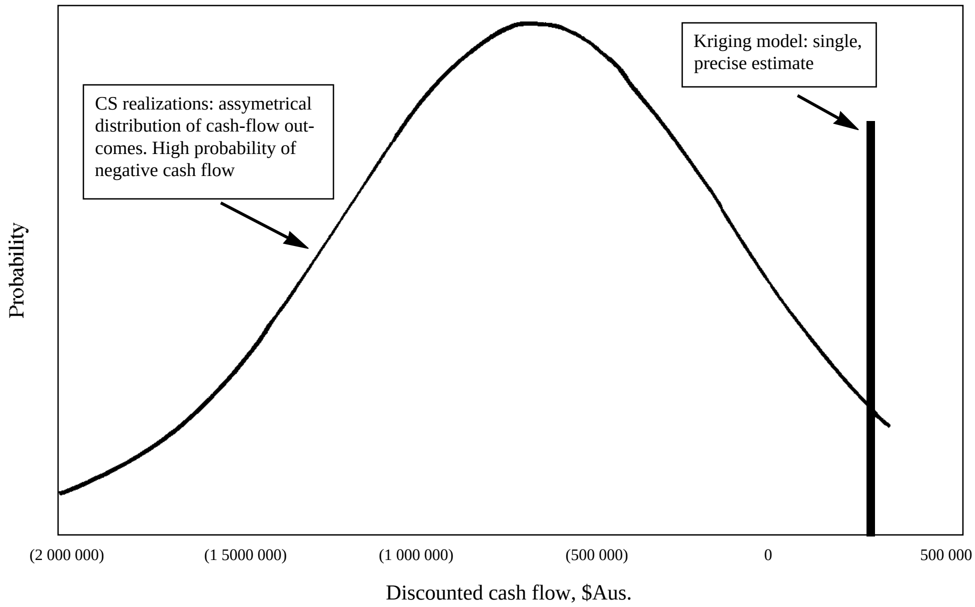


Fig. 11 Distribution of DCF outcomes in period 10 of mine production schedule and comparison with single estimate

The variation in DCF outcomes, shown in Fig. 10, highlights high-risk periods during the life of the project. The distribution of DCF for a single period, shown in Fig. 11, quantifies the risk of negative cash flows in the last stages of this production schedule. Having access to such information prior to mining is a valuable asset when determining such key parameters as the ultimate size of the project and the risk profile for that life.

Conclusions

Geological uncertainty as an element in key parameters of open-pit mining projects can be quantified by conditional simulation combined with open-pit optimization studies. Having an accurate assessment of uncertainty arising from grade variability in the ore reserve allows risk in a mining project to be quantified and considered in decision-making processes. This knowledge adds value to a project before the ore reserve is depleted and before development capital is committed to the project. Conditional simulation technologies provide some answers as to how well the project, and in particular, the orebody, is known. The challenge is to use currently available technologies, including conditional simulations, in a way suitable for operating mines, as well as to develop new technologies that advance the technical capability to model and quantify uncertainty and risk accurately. Future enhancements of today's simulation technologies should include methods that are computationally efficient and also able to simulate highly complex orebodies within the constraints of the industrial environment. An additional point to be stressed is that the traditional, non-risk-based, optimization approaches may not provide average assessments of key project indicators or truly optimal designs in the presence of geological uncertainty. In moving forward from the present optimization practices and their limitations, further technical integration of uncertainty in optimization algorithms is needed to enhance the interaction and efficacy of open-pit optimization and risk assessment.

Acknowledgement

R. Dimitrakopoulos thanks Steve Begg for introducing him to the 'real options' framework.

References

1. Lane K. Optimization: is it the best? In *Optimizing with Whittle, strategic mine planning* (Box Hill, Vict.: Whittle Programming, 1999), 1–7.
2. Lerchs H. and Grossmann L. Optimum design of open-pit mines. *Trans. CIM*, **LXVII**, 1965, 17–24.
3. Whittle J. Beyond optimization in open-pit design. In *Proc. Canadian Conf. Computer applications in the mineral industries*, (Rotterdam: Balkema, 1988), 331–7.
4. Whittle J. A decade of open-pit mine planning and optimization—the craft of turning algorithms into packages. In *APCOM '99* (Golden: Colorado School of Mines, 1999), 15–24.
5. Rendu J-M. Geostatistical simulations for risk assessment and decision making: The mining industry perspective. In Int. Symp. geostatistical simulations mining: uncertainty models, risk analysis and optimization of mining operations. CD, Bryan Research Centre, Brisbane, Australia, 1999. ISBN 186 499 3855
6. Dowd P. Risk assessment in reserve estimation and open-pit planning. *Trans. Instn Min. Metall. (Sect. A: Min. industry)*, **103**, 1994, A148–54.
7. Rossi M.E. and Camacho J. V. Using meaningful reconciliation to evaluate predictive models. *Trans. Soc. Min. Met. Explor.*, **308**, 2000, 78–82.
8. Dimitrakopoulos R. Conditional simulation algorithms for modelling orebody uncertainty in open-pit optimization. *Int. J. Surface Mining Reclamation Environment*, **12**, no. 4, 1998, 173–9.
9. Dowd P. Risk in minerals projects: analysis, perception and management. *Trans. Instn Min. Metall. (Sect. A: Min. industry)*, **106**, 1997, A9–18.
10. Manfrino A. Impact of high risk grade zones in optimal pit limits: a sensitivity study in a gold deposit. In Int. Symp. geostatistical

simulations mining: uncertainty models, risk analysis and optimization of mining operations. CD, Bryan Research Centre, ISBN 186 499 3855, Brisbane, Australia, 1999.
11. Ravenscroft P. J. Risk analysis for mine scheduling by conditional simulation. *Trans. Instn Min. Metall. (Sect. A: Min. industry),* **101**, 1992, A104–8.
12. Smith M. and Dimitrakopoulos R. Influence of deposit modelling on mine production scheduling. *Int. J. Surface Mining Reclamation Environment*, **13**, 1999, 173–8.
13. Amram M. and Kulatilaka N. *Real options: managing strategic investment in an uncertain world* (Harvard: Harvard Business School Press, 1999), 246 p.
14. Moel A. and Tufano P. Bidding for the Altamina mine: valuation and incentives in a real options context. Harvard Business School, note 297-054/297-055, 1998.
15. Vallee M. Mineral resource + engineering, economic and legal feasibility = ore reserve. *CIM Bull.*, **90**, 2000, 53–61.
16. Rossi M. and Parker H. M. Estimating recoverable reserves—is it hopeless? In Dimitrakopoulos R. ed. *Geostatistics for the next century* (Dordrecht: Kluwer, 1994), 259–76.
17. Halton J. H. A retrospective and prospective survey of the Monte Carlo method. In Zemanian A. H. ed. *S.I.A.M. Review*, 1970, 1–63.
18. Dimitrakopoulos R. Orebody uncertainty, risk assessment and profitability in recoverable reserves, ore selection and mine planning. WH Bryan Mining Geology Research Centre, Brisbane, Australia, 2000.
19. Dimitrakopoulos R. and Luo X. Generalized sequential Gaussian simulation. *Math. Geol.*, 2002, in press.
20. Journel A. G. Modelling uncertainty: some conceptual thoughts. In Dimitrakopoulos R. ed. *Geostatistics for the next century* (Dordrecht: Kluwer, 1994), 30–45.
21. Verly G. Sequential Gaussian cosimulations: a simulation method integrating several types of information. In Soares A. ed. *Geostatistics Troia '92* (Dordrecht: Kluwer, 1993), 543–54.

Authors

R. Dimitrakopoulos has worked and taught in Australia, North and South America, Europe, the Middle East, South Africa and Japan. He is currently Professor and Director of the WH Bryan Mining Geology Research Centre, University of Queensland.

Address: WH Bryan Mining Geology Research Centre, The University of Queensland, Brisbane, Qld 4072, Australia; *e-mail* roussos@uq.edu.au

C. Farrelly has worked for many years in resource estimation, geological modelling and geostatistics. He is currently a Ph.D. student at the WH Bryan Mining Geology Research Centre, University of Queensland.

M. Godoy obtained B.Sc. and M.Eng. degrees at the Federal University of Rio Grande do Sul, Brazil, and is currently a Ph.D. student at the WH Bryan Mining Geology Research Centre, University of Queensland.